Time-Controlled Descent Guidance in Uncertain Winds

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A procedure has been developed for constructing a statistical model of the altitude-dependent mean wind profile from the climatological data at particular locations. The model is constructed by fitting a Markov process, with altitude as the stage variable, to historical wind statistics. The wind model, together with the aircraft dynamics and the error characteristics of the navigation system, are incorporated in the design of a state estimator, which gives the minimum variance estimate of the aircraft state and the wind vector. The state and wind estimates are used as inputs to a linear feedback law for guiding the aircraft along the nominal trajectory. An example design of a time-constrained (4D RNAV) descent guidance system is presented, showing tracking accuracy, control activity, and probability of arrival time with and without the wind estimator.

Nomenclature

K = feedback matrix, 8×3
Q = state error weighting matrix, 8×8
P = covariance matrix of wind sample process
R = control error weighting matrix, 3×3
u = control vector with elements π, α, ϕ_a
v = independent measurement noise, a six vector
v_a = airspeed, m/sec
w = independent input noise, a two vector
x = state vector with elements x_a, y_a, z_a, $\dot{x}_a$, $\dot{y}_a$, $\dot{z}_a$, x_{w1}, x_{w2}
$\hat{x}_a$ = aircraft state estimate, a six vector
$\hat{x}_w$ = wind model state estimate, a two vector
x_w = wind model state, a two vector
x_a = longitudinal position, m
x_{w1} = wind along x_a, m/s
x_{w2} = wind along y_a, m/s
$\dot{x}_a$ = longitudinal velocity, m/s
y = horizontal components of the wing velocity, m/s, a two vector
y_a = lateral position, m
$\dot{y}_a$ = lateral velocity, m/s
z = output or measurement vector with elements $x_{\dot{\alpha}}$, $y_{\dot{\alpha}}$, $z_{\dot{\alpha}}$, v_a, γ_a, ψ_a
z_a = normal position (perpendicular distance from flight path), m
$\dot{z}_a$ = normal velocity, m/s
γ_a = flight path angle, deg
ψ_a = heading angle, deg
π = thrust, kN
ϕ_a = bank angle, deg
χ_a = aircraft state vector, three positions, m; three velocities, m/s; in body axis
α = angle of attack, deg

I. Introduction

COMPUTERIZED onboard navigation systems are increasingly used to monitor and control flight performance and to maintain up-to-date estimates of such critical flight variables as landing time and fuel reserves. In current airline practice, reserve fuel and landing time, along with specification of the route, are first generated prior to takeoff by the airlines' flight planning systems. Such systems are highly automated and are therefore able to utilize detailed wind profile forecasts from the weather services to generate fuel-efficient routes for each flight.

However, uncertainties in forecasted winds and terminal area ATC delays often require aircraft to carry large quantities of reserve fuel, thus reducing the value of sophisticated flight planning. Moreover, precise knowledge of landing time (important in the selection of efficient flight schedules) will play a crucial role in the future air traffic control system. This system will minimize delays by assigning landing slots prior to takeoff for aircraft equipped with time-controlled area navigation systems (4D RNAV). Various deterministic approaches to this type of system are discussed in Refs. 1-3. Previous experience shows that its performance depends critically on accurate prediction and control of aircraft states in the presence of uncertain winds.

In this paper, the objective prediction of the wind profile from navigation system measurements onboard the airplane is considered, and the time-controlled descent guidance in uncertain winds is approached as a stochastic control problem. The design procedure developed here, specifically for descent guidance, is equally applicable to other flight segments and potentially can also include uncertainties in other meteorological variables (temperature and pressure).

Climatological wind statistics based on historical rawinsonde records for a specific area, augmented with the current ground-level wind measurement and the last rawinsonde record, are employed to generate a stochastic mean wind model. This is illustrated in Sec. II. In Sec. III, models for navigation system (VOR-DME) and air data errors are developed, and the aircraft equations of motion are linearized along a minimum-fuel descent trajectory. Data for the nominal trajectory are derived from Ref. 4. The stochastic models and dynamic equations define the structure of the estimator and controller. The control law, derived in Sec. IV, is obtained by choosing the best tradeoff between arrival time accuracy and aircraft state errors, on the one hand, and control activity, on the other, using probabilistic measures of these quantities. Finally, the performance of the descent guidance system designed is discussed in Sec. V.

II. Wind Model

The first important aerospace requirement for a statistical model of mean winds aloft came from the space launches, where such a model was required prior to the launch for control and structural performance analysis of the rocket during the ascent.[5] At the same time, correlations among wind components at different altitudes became available and were proposed for improved a posteriori wind estimation from measurements at different levels.[6]

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This idea is generalized here. The family of spatial correlations between altitudes enables one to model mean wind randomness as a Markov process, with altitude as an independent (or state) variable. Assuming horizontal homogeneity of the wind in the airspace around the airport and above the planetary boundary layer, and considering the process stationary in the interval of time required for the descent from the cuise altitude to the ground, the model can be applied to the analysis of the guidance of a commercial airplane on the landing approach. The mean wind process is defined by a random vector with two components, the southerly and the westerly, at intervals of 1 km of altitude from the ground to 27 km. A gaussian distribution is adopted to describe the mean wind statistics. Data for a particular area are obtained from rawinsondes launched two to four times a day from the major airports. Monthly or seasonal wind statistics of data collected over a period of years are used to compute the mean value and covariance matrix of the wind vector.[7]

It is known that the process is not precisely gaussian, since, among other nongaussian characteristics, the constant altitude marginal distribution functions are elongated in the direction of the mean wind. However, the gaussian assumption is desirable in order to obtain a model that can be used for practical filter design. Moreover, a filter based on the model built on the empirical second-order statistics is optimum without the gaussian assumption in that it minimizes the least-squares estimation error.

In addition to historical rawinsonde data, there are three additional data sources available for wind profile estimation: 1) wind profile measurements made by aircraft that have recently completed their flight; 2) wind profile measurements obtained from the most recent rawinsonde launch (at most, 12 hours ago); and 3) the present airport surface wind measurement. The use of the first two data sources to update the wind profile requires the knowledge of persistence defined as the correlation of wind measurements (at fixed altitudes) taken at different time instants. Knowledge of persistence is incomplete in the high-altitude airspace, though some data on temporal correlations are available.[8] By applying standard regression analysis to such data when available, the a posteriori distribution (mean and covariance) conditioned to the most recent wind profile measurements can therefore be constructed and can be adopted as the final representation of the wind profile from 1 km to 27 km at any instant of time.

The definition of a linear Markov model of the mean wind randomness from the process covariance matrix is known as the problem of realization. This problem consists of specifying a dynamic system, driven by independent noise, whose output realizes the given process covariance matrix. In the following, E and cov denote the expectation and covariance operators, respectively. The model is defined in state variable form, with altitude in intergral kilometers as the stage variable i (see Ref. 9 for a discussion of these models):

$$x_W(i-1) = A_W(i)\cdot x_W(i) + w_W(i) \tag{1a}$$

$$y_W(i) = C_W(i)\cdot x_W(i) \qquad 1 \le i \le 10 \tag{1b}$$

$$\mathrm{E}[x_W(10)] = 0 \qquad \mathrm{cov}[x_W(10)] = P_W(10) \tag{1c}$$

$$\mathrm{E}[w_W(i)] = 0 \qquad \mathrm{cov}[w_W(i)] = Q_W(i) \tag{1d}$$

where $y_W(i)$ is the two-component wind vector at level i; n is the model order; the dimensions of $x_W(i)$, $w_W(i)$, $A_W(i)$, and $C_W(i)$ are $n\times 1$, $n\times 1$, $n\times n$, $2\times n$, respectively, and w_W is an n-dimensional, independent white noise process. Several references are available on the exact realization problem when restrictive assumptions are imposed on the process covariance matrix.[10-12] The most common assumptions are finite dimensionality of the exact realization and stationary process. However, analysis of experimental covariance shows that, in the present case, classical techniques are not convenient. In fact, the process is not stationary in the altitude, and an exact realization would result in a too-high dimensional model. Therefore, a special lower-order model approximation is proposed.

A notable early study of this problem can also be found in Ref. 13. However, the present approach offers the advantages of greater flexibility and generality for modeling multivariable Markov processes. It permits the use of models of arbitrary order and includes an efficient algorithm for synthesizing the model on a digital computer.

Moreover, the procedure is chosen so that the resulting model features statistically guaranteed properties with respect to the real process. We shall explain this property here specifically for an airplane in descent flight. The airplane (a subsonic turbofan) will have begun its descent approximately from an altitude level i, 10 km, and is currently assumed to be at level i, $1\le i\le 10$. Thus, the onboard estimator-predictor has available wind measurements for the interval 10 to i, and its function is to use these to estimate the wind profile between levels i and 1. The statistically guaranteed property of the model requires that the covariance $\hat{P}_M(i)$ (dimensions $2i\times 2i$) of the wind estimation error computed from the model (using measurements collected in flying through levels 10 to i) is an upper bound of the real estimation error covariance $\hat{P}(i)$, $[\hat{P}_M(i)-\hat{P}(i)\ge 0]$, resulting from the actual process, when in both cases linear measurement channels with additive noise and linear estimators of identical characteristics are used. As a result, the closed-loop control design based on such a model also gives a predicted performance which is an upper bound of performance resulting from the actual process. A complete description of the procedure is given in Ref. 14. Here the general concepts are outlined.

Indicate with $P(i)$ and $P_M(i)$ of dimension $2(11-i)\times 2(11-i)$, $i=10, 9,\ldots,1$ the portions of covariance matrices of the wind sample process between the ith and the 10th level $[y_W'(i),\ y_W'(i+1),\ldots,y_W'(10)]'$, respectively, for the process and the model (1 km of altitude separation between levels). To satisfy the condition $\hat{P}_M(i)-\hat{P}(i)\ge 0$, $1\le i\le 10$, it is necessary and sufficient to verify (for a proof see Ref. 14)

$$P_M(1) - P(1) \ge 0 \tag{2}$$

Generating the covariance matrix of a model of fixed order n which satisfies Eq. (2) is solved by defining a sequence of decompositions for $P_M(1)$ and satisfying recursively the conditions

$$P(i) \le P_M(i) = \left[\begin{array}{c|c|c} I_{2\times 2} & K(i+1) & 0 \\ \hline 0 & \multicolumn{2}{c}{I_{2(10-i)\times 2(10-i)}} \end{array}\right] \times \left[\begin{array}{c|c|c} S(i+1) & R(i+1) & 0 \\ \hline R'(i+1) & \multicolumn{2}{c}{} \\ \cline{1-1} 0 & \multicolumn{2}{c}{P_M(i+1)} \end{array}\right] \times \left[\begin{array}{c|c|c} I_2 & K(i+1) & 0 \\ \hline 0 & \multicolumn{2}{c}{I_{2(10-i)\times 2(10-i)}} \end{array}\right]' \tag{3}$$

(where the prime (′) after a matrix designates the transpose operation) and

$$P(10) \le P_M(10) \tag{4}$$

with $K(\cdot)$, $S(\cdot)$, $R(\cdot)$, $P_M(10)$ design matrices of dimension, respectively, $2\times n$, 2×2, $2\times s$, 2×2 for a given n, and $s\le n-2$. If $n>2$ for values of i such that $2(10-i)<n$, then S,

Table 1 Second-order wind model-coefficient and covariance matrices for three of ten altitude levels

	9km	6km	2km
$A_W(i)$ coefficient matrix =	$\begin{bmatrix} 0.78 & 0.017 \\ 0.062 & 0.82 \end{bmatrix}$	$\begin{bmatrix} 0.82 & -0.013 \\ 0.056 & 0.82 \end{bmatrix}$	$\begin{bmatrix} 0.60 & -0.114 \\ -0.124 & 0.582 \end{bmatrix}$
$Q_W(i)$ input noise covariance matrix =	$\begin{bmatrix} 24.0 & -1.1 \\ -1.1 & 24.0 \end{bmatrix}$	$\begin{bmatrix} 14.8 & 0.22 \\ 0.22 & 14.1 \end{bmatrix}$	$\begin{bmatrix} 19.4 & 3.42 \\ 3.42 & 18.6 \end{bmatrix}$

$K(i)$, and $R(i)$ will have the dimensions $2\times 2(10-i)$. In the construction of the model those matrices will be bordered by zeros. Arbitrary models satisfying Eqs. (3) and (4) can be generated algorithmically. Then an approximation measure between model and process is defined as

$$J=\text{trace}\,[P_M(1)-P(1)] \tag{5}$$

which, using the sequential decomposition, can be put in the recurrence form

$$J=\sum_{i=10}^{2}\text{traces}\,[S(i)+K(i)\cdot R'(i)+R(i)K'(i)$$

$$+K(i)P_M(i)\cdot K'(i)]+\text{trace}\,[P_M(10)-P(10)] \tag{6}$$

The approximation model design is obtained by minimizing

$$J_{\text{opt}}=\min_{K(\cdot),R(\cdot),S(\cdot),P_M(10)} J \tag{7}$$

with constraints $P_M(i)-P(i)\geq 0$.

A simple approximate minimization technique can be obtained that takes advantage of the sequential decomposition of the performance index. Then the model dynamics matrices (1) can be computed from $P_M(1)$ (see Ref. 14).

The example of the present paper uses historical wind statistics measured at Cape Kennedy, for the month of January, from 1956-1963.[7] Using standard regression analysis, the current ground-level wind information is incorporated in the historical data, resulting in an a posteriori mean wind vector and covariance matrix. The mean of the wind vector and the standard deviations with and without ground-level wind measurements are plotted in Fig. 1. As expected, the fractional reduction in the standard deviation obtained by using current ground-level measurements decreases with increasing altitude. From the a posteriori covariance matrix, a second-order Markov model approximation ($n=2$) of these wind statistics for the range of altitudes between 1 km and 10 km was computed with the technique previously described. Three of the sequence of nine matrices $A_W(i)$, $Q_W(i)$ are given in Table 1. A measure of approximation of the process evaluated by the normalized performance, trace $[P_M(1)-P(1)]/\text{trace}\,[P(1)]$, is 0.09.

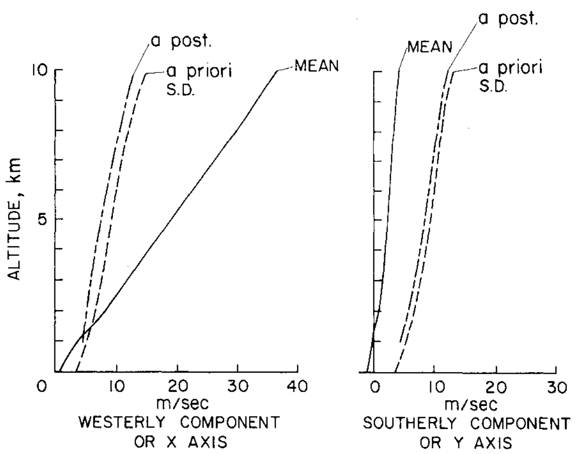


Fig. 1 Mean wind, a priori and a posteriori (from ground measurement) standard deviation for Cape Kennedy, month of January.

Table 2 Standard deviation of the measurement errors – VOR-DME transmitter located along X axis near the airport (errors are averaged for three equal distance sections of the flight)

Segment		I[a]	II	III
position inertial axis	x_{iner}, m	260	260	260
	y_{iner}, m	3300	2070	995
	z_{iner}, m	29.8	25.3	23.2
airspeed,	v_a, m/s	2.25	2.25	2.25
flight-path angle	γ_a, deg	1	1	1
heading angle	ψ_a, deg	1	1	1

[a] The first column also gives the initial condition uncertainties at the beginning of the descent.

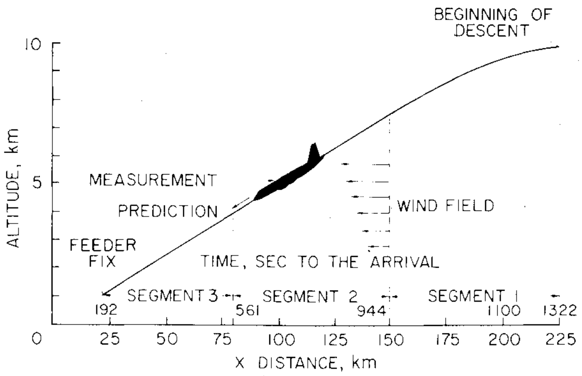
III. Nominal Descent Path, Measurement System, and Aircraft Dynamic Model

The design of the feedback system is based on the assumption that a nominal flight path and the nominal control settings are computed before the start of closed-loop control using the best estimate of the mean wind profile. The nominal flight path, speed, and thrust profiles chosen here to illustrate the design are given in Fig. 2. As previously stated, the nominal path is the minimum-fuel descent trajectory of a jet transport (given in Ref. 4). It begins at a 10-km altitude and ends at a 1-km altitude, covering a horizontal distance of 203 km in 1240 s.

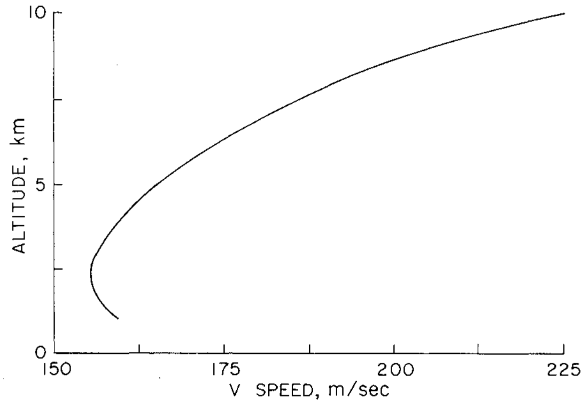
Because our interest here is centered on the effects of wind uncertainties, other sources of uncertainty (except those introduced by navigation and air data measurements) are neglected. Table 2 lists the quantities assumed to be measured onboard and specifies the standard deviation of the measurement errors. Measured outputs are the three inertial positions computed from VOR-DME signals, barometric altimeter, airspeed, flight-path angle, and heading, All errors are assumed to be statistically independent between successive measurements.

In deriving the aircraft dynamic equation for control system synthesis, it was assumed that the perturbations from the nominal flight path introduced by the mean wind uncertainties were small. This allows the sixth-order nonlinear aircraft point mass equations to be linearized along the nominal path.[15] All control and state variables used in the sequel are, therefore, perturbations from the nominal values. The controls are chosen as the thrust, angle of attack, and bank angle. The aircraft states are the three positions and the three inertial velocities resolved on the nominal aircraft body axes.

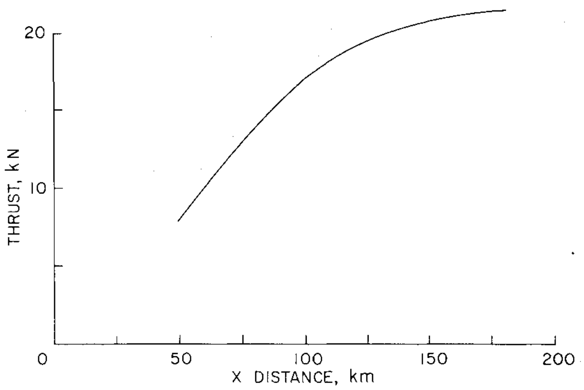
In general, coefficient matrices defining the perturbation equations are time-dependent, since they are a function of the nominal flight path. It was found, however, that three sets of time-invariant equations (each applicable over approximately



a) **Altitude vs distance**



b) **Altitude vs velocity**



c) **Thrust vs distance**

Fig. 2 Nominal minimum-fuel descent trajectory for modern tri jet, mass = 68000 kg, maximum sea-level thrust = 200 kN.

one-third of the nominal flight path) were sufficient for this study. The equations were then transformed to discrete-time, state-variable form as follows:

$$x_{j+1} = A_j x_j + B_j u_j + w_j \quad ,x_0 \tag{8a}$$

$$z_j = C_j x_j + v_j \qquad 0 \leq j \leq 124 \tag{8b}$$

The time discretization was chosen to be 10 s. The state vector x_j contains eight components, of which the first six are the aircraft states and the seventh and eighth are the two wind velocities. In order to add the wind states, the independent variable altitude of the wind model was transformed to time using the nominal trajectory. Table 3 gives the phugoid mode parameters and wind time constants of the three sets of linearized models, together with the altitudes of the nominal trajectory where each model is valid.

Other quantities appearing in Eq. (8) are as follows: the vector x_0 is a zero mean, random initial condition whose statistics are determined from a model of the navigation system used at the end of the cruise (first column of Table 2). The vector u_j is the input control vector of the airplane, w_j is a zero mean, independent input noise defined in the wind model, v_j are the zero mean independent noises modeling the navigation and the air data errors, and z_j is the measurement vector.

Table 3 Open-loop aircraft and mean wind dynamics along nominal trajectory in the $x\,z$ plane[a]

		Segment/altitude level		
		I/9 km	II/6 km	III/3 km
Aircraft phugoid mode	damping ratio	0.04	0.05	0.053
	period, s	91	77.6	71.3
Wind dynamic time constant, s		892.0	582.0	344.0

[a]Obtained transforming the wind model according to the nominal aircraft descent rate.

IV. Feedback Control System Design

The structure of the control system is chosen in the form of a linear state estimator operating on output measurements cascaded with a linear feedback law. The following design specifications are adopted:

1) The standard deviation of arrival time at the end point of the trajectory (1-km altitude) must fall within bounds required by air traffic control for efficient scheduling of landing time. Since the end point of the trajectory is not the runway, but a location known as the feeder fix, 23 km from touchdown, a two-sigma accuracy of 10 s is considered adequate for an advanced air traffic control system.

2) The standard deviation of errors from the nominal flight path should not exceed certain bounds. In the longitudinal, vertical, and lateral directions, these errors should be maintained within 1 km, 50 m, and 1 km, respectively. The standard deviation of airspeed error should be less than 8 m/s.

3) The standard deviation of control activity required to achieve specifications 1 and 2 must not result in excessive fuel consumption or passenger discomfort. Here, control of throttle activity as measured by the standard deviation from the nominal throttle position presents the most important constraint in achieving the specified arrival time accuracy.

For any linear estimator and feedback law, the standard deviations required to evaluate the design are derived from the model state covariance matrices Σ_j as a function of time, given by the solution of a difference matrix Lyapunov equation.[16] These a priori covariances of the state, and therefore also of the control, determine the effect of all uncertainties included in the model for the chosen estimator and feedback law.

To calculate the probability distribution of arrival time from the time history of the covariance matrix, the following approximate approach is taken. Indicate with σ_j the standard deviation of the horizontal position at instant j (obtained from the state covariance matrix Σ_j), with $\bar{x}_j$ the nominal horizontal position and with $\bar{x}_N$ the feeder fix, or final position. Probability that the airplane at instant j has already crossed the feeder fix is

$$p(x_j \geq \bar{x}_N) = 0.5 - \mathrm{erf}\left(\frac{\bar{x}_N - \bar{x}_j}{\sigma_j}\right) = q_j \tag{9}$$

Introduce the two-state family of random variables ξ_j with value one if the airplane at instant j has already crossed the feeder fix point, otherwise, zero. The variables ξ_j are obviously dependent, and probabilities of each of the two events

Table 4 Final design matrices[a]

Feedback matrices, K

Initial time							
−0.406E-01	−00.000	0.854E 01	−0.179E 03	−0.000	0.125E 02	0.612E 02	0.633E 00
0.981E-04	−00.000	−0.824E-03	0.114E 00	−0.000	0.682E-01	−0.766E-02	−0.579E-03
−0.000	− 0.9658-04	−0.000	−0.000	−0.240E-01	−0.000	−0.362E-03	−0.675E-02
Final time							
−0.216E 01	−0.000	0.580E 02	−0.970E 03	−0.000	0.413E 02	0.529E 03	−0.437E 02
0.447E-04	−0.000	0.667E-02	−0.396E-01	−0.000	0.697E-01	0.429E-01	0.216E-03
−0.000	−0.670E-03	−0.000	−0.000	−0.457E-01	−0.000	0.476E-04	−0.117E-02

Nonzero Q and R matrix entries

	Segment numbers I	II	III
q_{11}	10^{-4}	5×10^{-4}	5×10^{-3}
q_{22}	10^{-4}	10^{-4}	10^{-4}
q_{33}	1.0	1.0	1.0
q_{44}	100.	100.	100.
q_{47}, q_{74}	−99.9	−99.9	−99.9
q_{66}	11.3	11.3	11.3
q_{77}	100.0	100.0	100.0
q_{76}, q_{67}	1.7	1.7	1.7
r_{11}	10^{-2}	10^{-3}	10^{-4}
r_{22}	10^{-5}	5×10^{-2}	10^{-2}
r_{33}	10^{4}	10^{3}	100.

[a] All state and control variables are errors relative to nominal values.

are

$$p(\xi_j=1|\xi_{j-1}=1)=1 \tag{10a}$$

$$p(\xi_j=1|\xi_{j-1}=0)=q_j \tag{10b}$$

$$p(\xi_j=0|\xi_{j-1}=1)=0 \tag{10c}$$

$$p(\xi_j=0|\xi_{j-1}=0)=1-q_j \tag{10d}$$

The probability that the random arrival time is at some instant prior to or equal to j is given by $p(\zeta_j=1)$ where ζ_j is defined as

$$\zeta_j=\xi_1+\xi_2+\ldots\xi_j \tag{11}$$

Expression for $p(\zeta_j)$ is computed from Eqs. (10),

$$p(\zeta_j=1)=q_1+(1-q_1)\cdot q_2+\ldots\left(\prod_{h=1}^{j-1}(1-q_h)\right)\cdot q_j \tag{12}$$

The design of the system was achieved with the Linear Quadratic Gaussian (LQG) technique.[17] That is, the expected value of an integral quadratic performance index is minimized in the time interval between the instant, $i=1$, at which air traffic control gives its approval to initiate the descent and, $i=N+\Delta$ a short time after the nominal flight reaches the feeder fix:

$$J=E\left\{\sum_{j=1}^{N+\Delta}x_j^TQ_jx_j+u_{j-1}^TR_{j-1}u_{j-1}\right\}$$

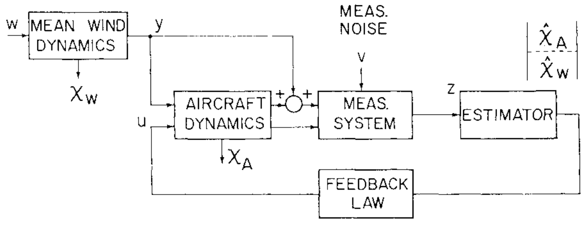


Fig. 3 Block diagram of estimator-controller.

It is known that the closed-loop solution of this problem is composed of 1) the minimum variance estimator of the state x_j and 2) a linear feedback law operating on the estimated state, which is the solution of the deterministic optimum control problem and is independent of the noise, but only a function of the weighting matrices Q_j and R_j.

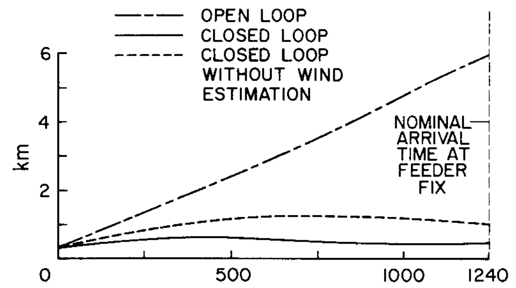
In the LQG design technique, the essential problem is choosing the weighting matrices Q_j and R_j that will result in a system meeting the performance specifications. This problem is most often solved by a trial-and-error process. Familiarity with the dynamics of the process being controlled, together with experience in using the LQG technique, helps to reduce the number of trial-and-error iterations required to obtain a satisfactory design.

In the present case, the first trial consisted of evaluating the open-loop performance of the system [Eqs. (8)]. Then the penalty on the controls R_j was reduced in steps to observe the effect of feedback on performance. When the variances of some of the state variables were within range of the target values, the relative weighting of the states, as determined by the ratios of diagonal terms in Q_j was iterated until all state variable variances were satisfactory. Finally, the Q_j matrix was made a function of j such that the weighting of state errors increased as j approached the last value ($j=N+\Delta=134$). This has the effect of reducing tracking accuracy and also control activity at the start of the trajectory when the aircraft is still far from the feeder fix. A satisfactory design was achieved after 20 iterations.

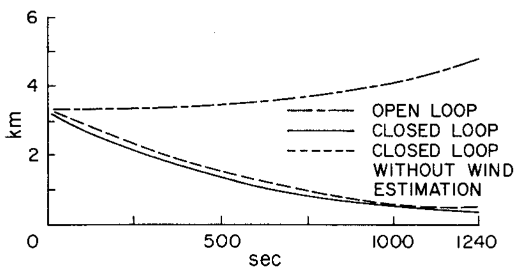
Parameters for the final design are given in Table 4. Figure 3 is a block diagram of the complete system, consisting of the wind model, aircraft model, measurement system, estimator, and feedback law. The feedback law parameters are given in Table 4 only for the initial and final time.

V. Discussion of Results

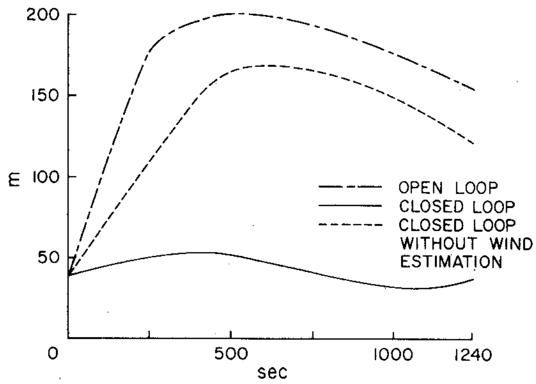
The matrix Lyapunov equation was solved for the evolution of the covariance matrices required to evaluate the performance of the design. From these, the standard deviations of the most important state and control variables were calculated and then plotted vs time in Fig. 4. Also, the standard deviations [together with Eq. (12)] were used to calculate the distribution of arrival time at the feeder fix and



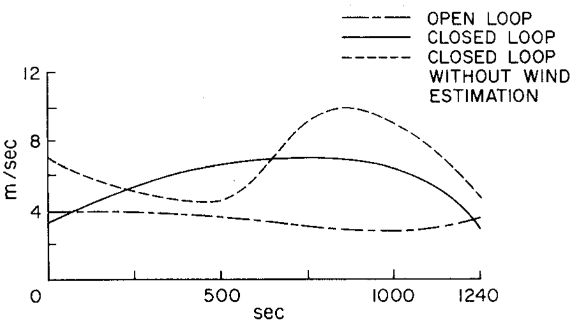
a) *x* standard deviation



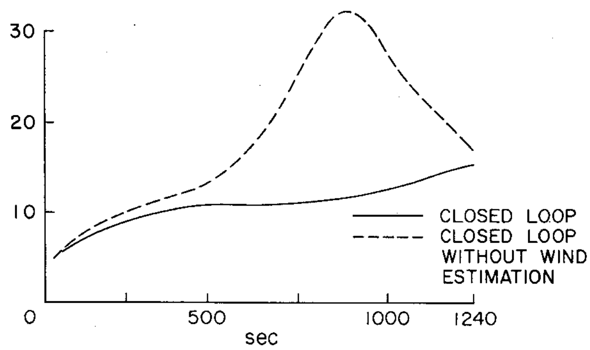
b) *y* standard deviation



c) *z* standard deviation, in percent of nominal thrust

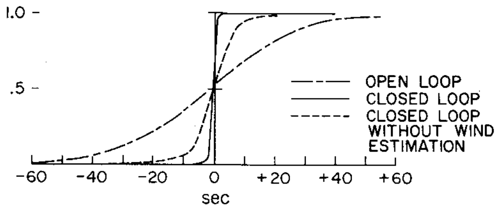


d) Airspeed standard deviation

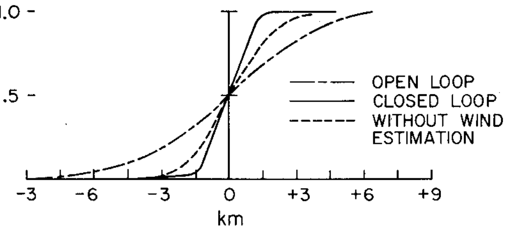


e) Throttle standard deviation, in percent of nominal thrust

Fig. 4 Standard deviation of guidance errors.



a) Probability that the airplane has crossed the feeder fix horizontal position at or before τ seconds from the nominal time



b) Probability that the airplane reaches 1 km of altitude at or before τ km from the feeder fix

Fig. 5 Time and altitude errors at the feeder fix.

the horizontal distribution of crossing the 1-km altitude at the feeder fix. These are plotted in Fig. 5. For comparison, the performance of the open-loop system without feedback and wind estimates is also shown in Figs. 4 and 5.

Analysis of the results shows that the closed-loop system with wind estimator meets all accuracy criteria in position and arrival time. However, the increased accuracy in these quantities is achieved at the cost of throttle and increased airspeed deviations from the nominal values compared to the open-loop descent. Airspeed deviations have approximately doubled, reaching a maximum standard deviation of 7 m/s. They are, however, below the design limit of 8 m/s. Throttle deviations have increased from zero in the open-loop descent to an average of approximately 1.5 kN in the closed-loop system. These deviations can still be considered low, and therefore acceptable, since they are only 10% of the nominal thrust used during the descent.

The arrival time accuracy for the closed-loop system, read from Fig. 5a, is approximately ± 8 s, with probability of at least 0.95. This compares to ± 50 s at 0.95 probability for the open-loop descent.

In another case studied, no significant deteriorations of the performances of Figs. 4 and 5 have been observed when flight-path angle and heading angle are not included in the measurements.

The errors of the closed-loop descent without feedback of wind estimates are approximately twice those with wind feedback. However, the most serious effect is in increased throttle activity, which has more than doubled in the middle part of the flight. In the attempt to satisfy the terminal performance on the guidance and time errors, the feedback gains increase in the second portion of the flight. Elimination of the wind statistic from the filter reduces its effectiveness and results in high levels of disturbance which are transmitted through the feedback loop to the input.

The data in Fig. 5b can be used to establish a rational choice for the minimum buffer distance between the feeder fix and the start of final approach, i.e., the outer marker of the Instrument Landing System. With closed-loop descent guidance, the buffer distance can be reduced from 7 km to 1.5 km.

VI. Conclusions

A procedure has been developed for constructing a statistical model of the altitude-dependent mean wind profile from the historical record of wind measurements at particular locations. The model is constructed by fitting a Markov process, with altitude as the stage variable, to the historical wind data. With respect to other approaches, the present model offers the advantages of a statistically guaranteed behavior with respect to the process. Moreover, the design algorithm appears especially convenient when ill-defined (not well behaved) experimental covariances are available. It also offers the possibility, when it is desired, to combine different statistics (seasonal, local) for the synthesis of a unique model to approximate each of these conditions. The wind model, together with the aircraft dynamics and the error characteristics of the navigation system are incorporated in the design of a state estimator, which gives the minimum variance estimate of the aircraft state and the wind vector. The state and wind estimates are used as inputs to a linear feedback law for guiding the aircraft along the nominal trajectory. Since the stochastic and dynamic models, and the estimator and the feedback law are linear, the performance analysis of the design does not require Monte Carlo simulation, but only the solution of a finite-difference covariance matrix equation. The procedure is applicable to aircraft guidance system design generally, but is of greatest relevance to those designs where uncertain winds and navigation errors can result in excessive control activity.

In the example design of a time-constrained (4D RNAV) descent guidance system, the use of the wind model increased arrival time and path tracking accuracy and decreased throttle activity by a factor of 2 compared to a design not using the wind model. It appears that arrival time accuracy at the feeder fix of ± 8 s (0.95 probability) for an aircraft starting from a point 200-km distant and at a 10-km altitude is possible with low throttle activity using the wind model and estimator described herein. This compares to an accuracy of ± 50 s (0.95 probability) for the same trajectory flown with conventional techniques and constant throttle setting.

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